

Level density for deformations of the Gaussian orthogonal ensemble

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Formulas are derived for the average level density of deformed, or transition, Gaussian orthogonal random matrix ensembles. After some general considerations about Gaussian ensembles, we derive formulas for the average level density for (i) the transition from the Gaussian orthogonal ensemble (GOE) to the Poisson ensemble and (ii) the transition from the GOE to m GOEs.

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I. INTRODUCTION

Deformed random matrix ensembles were introduced by Rosenzweig and Porter [1,2] to classify the conservation of electronic spin and orbital angular momentum in the spectra of complex atoms. Many varieties of deformed ensembles have since been constructed. Recent reviews of deformed ensembles (they are also called transition ensembles) can be found in Refs. [3,4]. Earlier results on deformed ensembles were reviewed in Ref. [5]. Among the applications they have found, we mention the breaking of time-reversal invariance [6] and the breaking of symmetries [7].

In this paper, we are concerned with the level density for deformed Gaussian orthogonal random matrix ensembles. Although the level density of the standard random matrix ensembles is not directly related to the physical many-particle level density, it is essential to the proper unfolding of fluctuation measures [8]. Unfolding is a transformation which leaves sequences of energy levels with a constant average density. Normally, experimental energy levels *and* the energy levels obtained from random matrix theory are unfolded (which removes the secular energy dependences) before fluctuation measures for both are calculated and compared. While the level densities of most random matrix ensembles are different, the behavior of the fluctuation measures of a wide class of ensembles is the same and hence denominated universal. For instance, each of the non-Gaussian ensembles studied in Ref. [9] is shown to have a characteristic average level density, however after unfolding, the nearest-neighbor spacing distributions and number variances of all are found to be identical to those of the standard Gaussian random matrix ensembles [10].

In the following, we derive formulas for the level density of two deformations of the Gaussian orthogonal ensemble (GOE). The first describes a transition from the GOE to an ensemble with Poisson fluctuation statistics, that is, from the GOE to an ensemble of diagonal matrices whose elements are independently Gaussian distributed [11,12]. Such random matrix models are of interest to the analysis of experimental data [13] and of dynamical models [14] whose fluctuation properties are intermediate between Poisson and GOE. The second deformation describes the transition from the GOE to m GOEs, that is, from the GOE to a block diagonal matrix with m blocks each of which is a GOE [15–17]. If the blocks are labeled by quantum numbers such as angular momentum [1] or isospin [18], then the transition ensemble classifies

their conservation or nonconservation. The m GOE to GOE transition is also relevant to the analysis of symmetry breaking in quartz blocks [19–21].

Our method extends that of Wigner [22], who showed, by assuming that terms containing patterns of unlinked binary associations dominate the averages of the traces of powers of matrices, that the level density for certain random matrix ensembles could be simply expressed as a Fourier of transform (see also Sec. III D of Ref. [5]). Previous results for the level density of deformed GOEs have been derived using Stieltjes transform methods [23,24]. These methods are in fact general enough to treat deformations of, and interpolations between, any class of matrix ensemble. However, the formulas obtained in the present paper have the advantage that they are explicit and simple to evaluate numerically. Other methods for calculating the level density [25,26] are restricted to deformations of the Gaussian unitary ensemble (GUE).

II. INTERPOLATING GAUSSIAN ENSEMBLES

The joint probability distribution of the matrix elements of a matrix, H , for an interpolating Gaussian ensemble may be expressed as

$$P(H,A,B) = Z^{-1}(A,B) \exp[-(A \text{tr} H^2 + B \text{tr} H_1^2)], \quad (1)$$

where Z is a normalization factor and $\text{tr} H$ denotes the trace of H . The structure of the matrix H_1 is chosen in such a way that it defines a subspace of H . The parameters A and B define the (energy) scale and degree of deformation. When $B \rightarrow 0$, the joint distribution becomes

$$P(H,A,0) = Z^{-1}(A,0) \exp(-A \text{tr} H^2). \quad (2)$$

If we assume that H is real symmetric, then the variances are given by

$$\overline{H_{ij}^2} = \frac{1 + \delta_{ij}}{4A}, \quad (3)$$

so that the limit $B \rightarrow 0$ defines the GOE. When $B \rightarrow \infty$, the elements of H_1 vanish and H is projected onto a sparse matrix, H_0 , whose elements are on the complementary subspace of H_1 . Therefore, the random matrices generated by Eq. (1) are the sum of two terms,

$$H = H_0 + H_1, \quad (4)$$

with the variances of the matrix elements of H_0 given by the right-hand side of Eq. (3) and those of H_1 given by

$$\overline{H_{1ij}^2} = \frac{1 + \delta_{ij}}{4(A+B)}. \quad (5)$$

This shows that when B goes from zero to infinity, the ensemble undergoes a transition from GOE to the Gaussian ensemble of sparse matrices defined by the choice of the structure of H_1 or, equivalently, of H_0 . It is also instructive to introduce the parameter

$$\alpha = (1 + B/A)^{-1/2}, \quad (6)$$

which measures the relative strength of H_1 and H_0 . The transition from $\alpha=0$ to $\alpha=1$ corresponds to the transition from $B=\infty$ to $B=0$.

III. LEVEL DENSITY FOR THE GOE

We now proceed to give a derivation of the semicircle law, valid for the GOE, which is equivalent to Wigner's for the ensemble of random sign symmetric matrices [22]. In general, the average level density (ALD) may be written as a Fourier transform,

$$\rho(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk F(k) e^{-ikE}, \quad (7)$$

where

$$F(k) = \int_{-\infty}^{\infty} dE \rho(E) e^{ikE} \quad (8)$$

$$= \sum_{n=0}^{\infty} \frac{(ik)^n}{n!} \int_{-\infty}^{\infty} dE \rho(E) E^n. \quad (9)$$

Let E_k denote the eigenvalues of an $N \times N$ matrix H , which satisfies Eqs. (2) and (3). From the exact expression,

$$\rho(E) = \frac{1}{N} \sum_{k=1}^N \overline{\delta(E - E_k)}, \quad (10)$$

for the ALD, one obtains the following connection between the moments of the eigenvalue distribution and the moments of the matrix elements,

$$\overline{E^n} \equiv \int_{-\infty}^{\infty} dE \rho(E) E^n = \frac{1}{N} \overline{\text{tr} H^n}. \quad (11)$$

Substituting Eq. (11) into Eq. (9), we obtain

$$F(k) = \frac{1}{N} \sum_{n=0}^{\infty} \frac{(ik)^n}{n!} \overline{\text{tr} H^n}. \quad (12)$$

It is also useful to note that by differentiating Eq. (8), the moments of the eigenvalue distribution can be expressed as derivatives of the Fourier transform of the ALD evaluated at zero, that is,

$$\overline{E^n} = i^{-n} F^{(n)}(0), \quad (13)$$

where $F^{(n)}(0) = [d^n F / dk^n]_{k=0}$.

For large matrices, $N \gg 1$, the average of the trace is dominated by the terms in which matrix indices may be contracted in such a way that we are left with $s=n/2$ pairs of matrix elements. This means that we can write

$$\overline{\text{tr} H^n} \simeq C_s N^{s+1} \sigma^{2s}, \quad (14)$$

where we have introduced the notation

$$\sigma^2 = \frac{1}{4A} \quad (15)$$

for the variance of the off-diagonal matrix elements. The factor C_s counts the number of contractions that leads to pairs. We observe that in a contraction, $s-1$ indices are eliminated and $s+1$ remain, which explains the power of N in the above expression and allows us to write

$$C_s = \frac{1}{s} \frac{(2s)!}{(s-1)!(s+1)!}, \quad (16)$$

where the binomial factor counts the number of ways $s+1$ indices can be extracted out of $2s$ ones. The factor is then divided by s , that is, the number of ways $s-1$ indices can be eliminated without changing the contraction.

Substituting Eq. (14) in Eq. (12), we find that the Fourier transform of the average density is given by

$$F_1(a, k) \simeq \frac{1}{ka/2} \sum_{s=0}^{\infty} \frac{(-)^s (ka/2)^{2s+1}}{s!(s+1)!} = \frac{1}{ka/2} J_1(ka), \quad (17)$$

where $J_1(x)$ is a first-order Bessel function and

$$a = \sqrt{N/A}. \quad (18)$$

Using the formula

$$(1-x^2)^{1/2} = \frac{1}{2} \int_{-\infty}^{\infty} y^{-1} J_1(y) e^{-ixy} dy, \quad (19)$$

the Fourier transform in Eq. (7) may be evaluated to obtain the semicircle law,

$$\rho_1(a, E) = \begin{cases} \frac{1}{\pi a^2/2} \sqrt{a^2 - E^2}, & |E| \leq a, \\ 0, & |E| > a, \end{cases} \quad (20)$$

where the radius of the semicircle is given by Eq. (18). The cumulative level density (CLD), $x(E)$, is defined by

$$x(E) = \int_0^E \rho(E') dE', \quad (21)$$

which for the semicircle law is found to be

$$x_1(a, E) = \begin{cases} -1/2, & E < a, \\ \frac{E\sqrt{a^2 - E^2} + a^2 \arcsin(E/a)}{\pi a^2}, & |E| \leq a, \\ 1/2, & E > a. \end{cases} \quad (22)$$

The average number of levels up to energy E is given in terms of the CLD by $N[x(E) - x(-\infty)]$.

IV. THE TRANSITION GOE TO POISSON

Before considering the transition from Poisson to GOE, we derive a formula for the ALD for a more general case.

A. Level density for $H_0 + H_1$

The trace of the n th power of Eq. (4) can be written as

$$\text{tr}H^n = \text{tr}(H_0 + H_1)^n = \sum_{l=0}^n \frac{n!}{l!(n-l)!} \text{tr}H_0^l H_1^{n-l}. \quad (23)$$

If H_0 and H_1 are statistically independent, we can write

$$\overline{\text{tr}H_0^l H_1^{n-l}} = \sum_j \overline{E_{0j}^l} \overline{\langle j|H_1^{n-l}|j\rangle} \quad (24)$$

$$= \overline{E_0^l} \overline{\text{tr}H_1^{n-l}}, \quad (25)$$

where E_{0j} and $|j\rangle$ are defined by the eigenvalue equation $H_0|j\rangle = E_{0j}|j\rangle$ and $\overline{E_0^l}$ denotes the l th moment of the eigenvalue distribution of H_0 . Equations (23) and (25) allow us to write [cf. Eq. (11)]

$$\frac{1}{N} \overline{\text{tr}H^n} \approx \int_{-\infty}^{\infty} dE_0 \rho_0(E_0) \int_{-\infty}^{\infty} dE_1 \rho_1(E_1) (E_0 + E_1)^n, \quad (26)$$

where ρ_0 and ρ_1 are the average level densities corresponding to H_0 and H_1 , respectively. Equation (26) is only approximate because in resumming the binomial series we have kept linked as well as unlinked binary associations of H_0 and H_1 . In the following subsection, we show that for the case of the transition GOE-Poisson the resulting discrepancy in the ALD is small. We also show how to correct for this discrepancy.

Substituting Eq. (26) into Eq. (12), we find

$$F(k) = \int_{-\infty}^{\infty} dE_0 \rho_0(E_0) \int_{-\infty}^{\infty} dE_1 \rho_1(E_1) e^{ik(E_0 + E_1)} = F_0(k) F_1(k), \quad (27)$$

where F_0 and F_1 are the Fourier transforms of ρ_0 and ρ_1 , respectively. The ALD is obtained by substituting Eq. (27) into Eq. (7). Another representation is obtained by noting that since Eq. (27) is a product of Fourier transforms, the ALD of H is given by the convolution of the average level densities of H_0 and H_1 ,

$$\rho(E) = \int_{-\infty}^{\infty} dE' \rho_0(E') \rho_1(E - E'). \quad (28)$$

The only assumption required to derive Eqs. (27) and (28) is that the matrix elements of H_0 and H_1 be statistically independent.

B. Level density for the transition GOE to Poisson

To specialize the results of the last subsection to the transition GOE-Poisson, H_0 is chosen to be the diagonal matrix $H_{0ij} = E_{0i} \delta_{ij}$ whose eigenvalues, E_{0i} , are independent random variables with Gaussian distribution

$$\rho_0(E) = \sqrt{\frac{A}{\pi}} e^{-AE^2}. \quad (29)$$

The variance of H_0 is thus

$$\overline{H_{0ij}^2} = \frac{\delta_{ij}}{2A} = a^2 \frac{\delta_{ij}}{2N}. \quad (30)$$

We choose H_1 to be a diagonal-less matrix whose matrix elements are independent Gaussian variables with zero mean and variances

$$\overline{H_{1ij}^2} = \alpha^2 \frac{1 - \delta_{ij}}{4A} = \alpha^2 a^2 \frac{1 - \delta_{ij}}{4N}. \quad (31)$$

The ALD for H_0 alone is given by Eq. (29) and the CLD, Eq. (21), by

$$x_0(E) = \frac{1}{2} \text{erf}(\sqrt{AE}). \quad (32)$$

The Fourier transform of Eq. (29) is

$$F_0(k) = e^{-k^2/4A}. \quad (33)$$

Note that for fixed a , $A \rightarrow \infty$ as $N \rightarrow \infty$, so that

$$\rho_0(E) \xrightarrow{N \rightarrow \infty} \delta(E). \quad (34)$$

The ALD for H_1 alone is given by $\rho_1(\alpha a, E)$, that is, by the semicircle law [see Eq. (20)] with the radius modified ($a \rightarrow \alpha a$) in accordance with Eq. (31) for the variance of H_1 [cf. Eq. (3)]. Its Fourier transform is $F_1(\alpha a, k)$ [see Eq. (17)]. The ALD for H_1 is given by the semicircle law in spite of the missing diagonal because off-diagonal matrix elements dominate (for N large enough).

Using Eqs. (27) and (7), the ALD which interpolates between the Gaussian and the semicircle as α varies between 0 and 1 is then found to be

$$\rho_\alpha(E) = \frac{1}{2\pi} \int_{-\infty}^{\infty} dk F_0(k) F_1(\alpha a, k) e^{-ikE} \quad (35)$$

$$= \frac{2\sqrt{A}}{\pi\lambda} \int_0^\infty \frac{dx}{x} e^{-x^2/4\lambda^2} J_1(x) \cos \frac{\sqrt{A}x E}{\lambda}, \quad (36)$$

where we have changed the integration variable to $x = \alpha ka$ and introduced the parameter

$$\lambda = \alpha \sqrt{N}. \quad (37)$$

In Fig. 1, we compare Eq. (36) for $\rho_\alpha(E)$ with histograms constructed by numerically diagonalizing random matrices for several values of α (see figure caption for details). It is seen that good agreement is obtained, there being, however, a small discrepancy in the transition region which is espe-

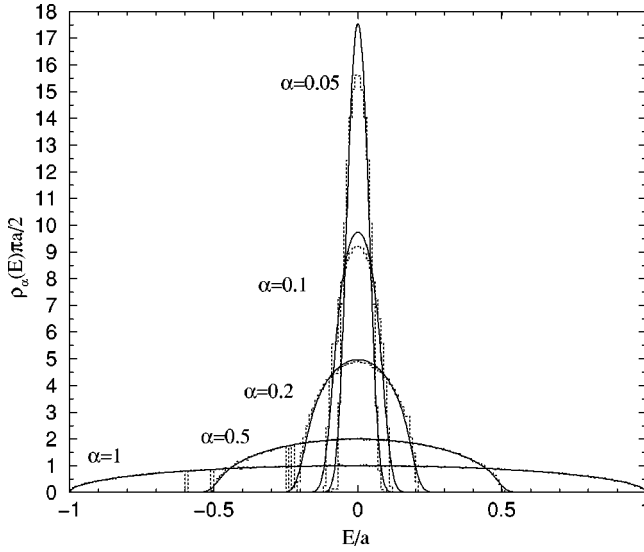


FIG. 1. Graph of $\rho_\alpha(E)$ for several values of α with $N=1000$. The calculations were performed with $A=N/4$ for which the radius of the Wigner semicircle is $a=2$. The solid lines were calculated using Eq. (36) and the dotted histograms by numerically diagonalizing an ensemble of 100 matrices.

cially apparent in the graph for $\alpha=0.05$. The CLD, Eq. (21), may be expressed using Eq. (36) as

$$x_\alpha(E) = \frac{2}{\pi} \int_0^\infty \frac{dx}{x^2} e^{-x^2/4\lambda^2} J_1(x) \sin \frac{\sqrt{Ax}E}{\lambda}. \quad (38)$$

This equation provides a more accurate manner of unfolding fluctuation measures than the polynomial unfolding used in Ref. [27]. Equation (38) is plotted for several values of α in Fig. 2.

The ALD may be expressed alternatively using the convolution formula, Eq. (28). In particular, when $N \rightarrow \infty$ for

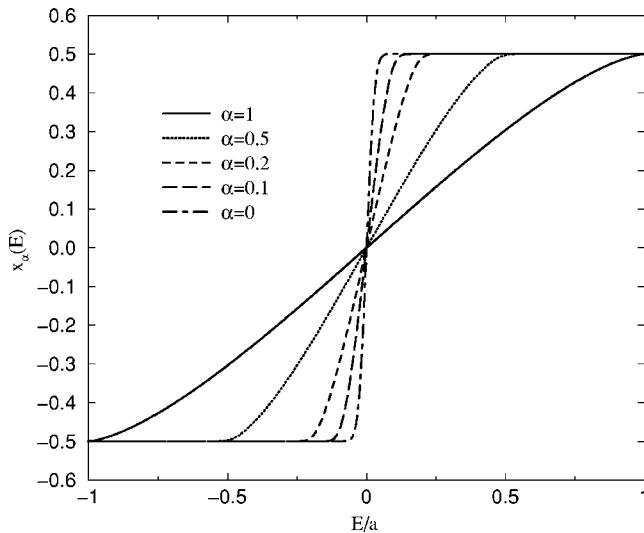


FIG. 2. Graph of $x_\alpha(E)$ calculated using Eq. (38) for several values of α with $N=1000$. The calculations were performed with $A=N/4$, for which the radius of the Wigner semicircle is $a=2$.

fixed α and a , we find from Eq. (28) and Eq. (34) that

$$\rho_\alpha(E) \xrightarrow{N \rightarrow \infty} \rho_1(\alpha a, E). \quad (39)$$

For finite N , Eq. (28) yields

$$\rho_\alpha(E) = \frac{2\sqrt{N}}{\pi^{3/2} a^3 \alpha^2} \int_{-\alpha a + E}^{\alpha a + E} e^{-NE'^2/a^2} \sqrt{\alpha^2 a^2 - (E - E')^2} dE'. \quad (40)$$

It is clear from these explicit formulas that the transition parameter is $\alpha^2 N$, as was already argued in the original paper of Rosenzweig and Porter [1]. The model described in this section can be cast in the form

$$H = H_0 + \frac{\lambda}{N^c} H_1, \quad (41)$$

with $c=1/2$, by modifying the definitions of the variances, Eqs. (31). Reference [26] considered the statistics of ensembles of the form of Eq. (41) for arbitrary c . In particular, the asymptotic behavior of the ALD as $N \rightarrow \infty$ depends critically on c , Eq. (39) being valid only for the special case $c=1/2$.

Equation (36) may be improved by comparing the resulting lowest moments of the eigenvalue distribution with the average trace of the corresponding powers of the Hamiltonian. Considering the latter first, we have

$$\overline{\text{tr}H^2} = \overline{\text{tr}H_0^2} + \overline{\text{tr}H_1^2} = N\overline{E_0^2} + C_1 N^2 \frac{\alpha^2 a^2}{4N} = \frac{N}{4A} (\lambda^2 + 2) \quad (42)$$

and

$$\begin{aligned} \overline{\text{tr}H^4} &= \overline{\text{tr}H_0^4} + 4\overline{\text{tr}H_0^2 H_1^2} + \overline{\text{tr}H_1^4} \\ &= N\overline{E_0^4} + 4\overline{E_0^2} C_1 N^2 \frac{\alpha^2 a^2}{4N} + C_2 N^3 \frac{\alpha^4 a^4}{16N^2} \\ &= \frac{N}{8A^2} (\lambda^4 + 4\lambda^2 + 6). \end{aligned} \quad (43)$$

Inserting Eq. (36) into Eq. (13) for the moments of the eigenvalue distribution, we find that

$$\overline{E^2} = \frac{1}{4A'} (\lambda'^2 + 2), \quad (44)$$

$$\overline{E^4} = \frac{1}{8A'^2} (\lambda'^4 + 6\lambda'^2 + 6). \quad (45)$$

Note that the coefficient of λ'^2 in Eq. (45) is 6. This is a result of the use of Eq. (26), which included linked binary associations of H_0 and H_1 . Equation (11) demands that

$$\overline{E^2} = \frac{1}{N} \overline{\text{tr}H^2}, \quad (46)$$

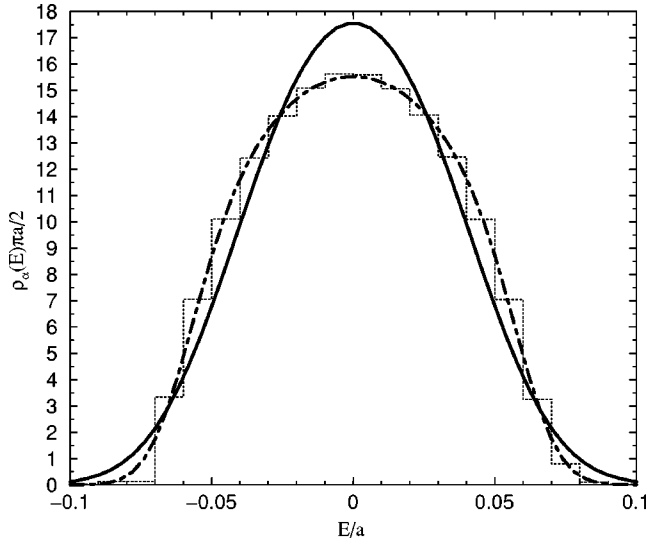


FIG. 3. Graph of $\rho_\alpha(E)$ showing the improvement obtained by demanding that the second and fourth moments of the eigenvalue distribution be exact for $\alpha=0.05$ and $N=1000$. The solid lines result from using Eq. (36) and the dot-dashed lines from using the same equation modified in accordance with Eqs. (48) and (49). The dotted histograms were obtained by numerically diagonalizing an ensemble of 100 matrices.

$$\overline{E^4} = \frac{1}{N} \overline{\text{tr} H^4}. \quad (47)$$

Solving Eqs. (46) and (47) for A' and λ' , we see that Eq. (36) will give the second and fourth moments of the eigenvalues distribution correctly if we make the substitution

$$\frac{1}{A} \rightarrow \frac{1}{A'} = \frac{1}{A} \left(1 + \lambda^2 \left[1 - \sqrt{1 + \frac{4}{\lambda^2}} \right] \right), \quad (48)$$

$$\lambda^2 \rightarrow \lambda'^2 = \lambda^2 \frac{\sqrt{1 + \frac{4}{\lambda^2}}}{1 + \lambda^2 \left[1 - \sqrt{1 + \frac{4}{\lambda^2}} \right]}. \quad (49)$$

In Fig. 3, we show the improvement to the ALD obtained by using Eqs. (48) and (49) in Eq. (36) for the worst case of Fig. 1 ($\alpha=0.05$). We see that the modified formula gives the ALD essentially exactly.

In Fig. 4, we show the peak value of the density of states as a function of λ (see figure caption). We see that both the modified and unmodified versions of Eq. (36) agree at the $\lambda=0$ limit. We also see that both versions of Eq. (36) reach the $N \rightarrow \infty$ limit [Eq. (39)] at roughly the same value of λ ($\lambda \approx 6$). However, the two versions deviate from each other in the transition region, the largest difference occurring around $\lambda \approx 1.5$ ($\alpha \approx 0.05$ for $N=1000$). Also shown for comparison is the interpolation formula for the ALD of Persson and Åberg [28], ρ_α^{PA} , Eq. (50). Finally, we show $\rho_\alpha(0)$ calculated using Eq. (36) modified in accordance with Eqs. (48) and (49).

$$\rho_\alpha^{\text{PA}} = \frac{N^{1/2}}{4\alpha N^{1/2} + 7N^{-1.5\alpha}}. \quad (50)$$

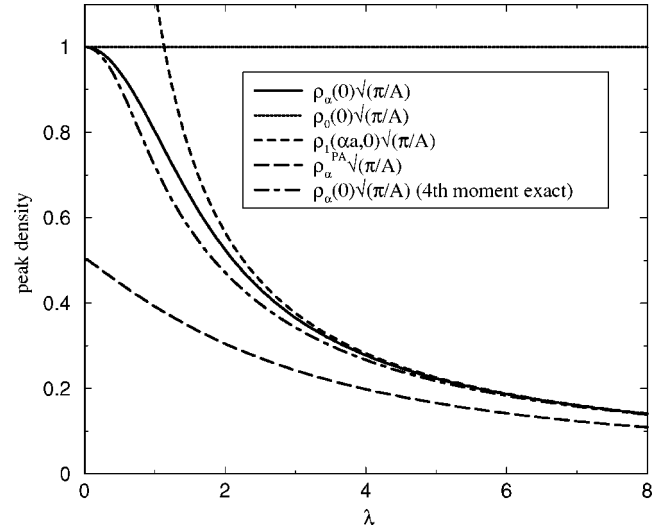


FIG. 4. Graph of the peak values of the density $\rho_\alpha(0)$, Eq. (36), as a function of the transition parameter λ , Eq. (37). For comparison we also show the limiting cases $\rho_0(0)$, Eq. (29), and $\rho_1(\alpha\alpha, 0)$, Eq. (39), as well as an interpolation formula given by Persson and Åberg [28] ρ_α^{PA} , Eq. (50). Finally, we show $\rho_\alpha(0)$ calculated using Eq. (36) modified in accordance with Eqs. (48) and (49).

V. THE TRANSITION GOE TO m GOES

We now calculate the ALD for the transition from the GOE to a superposition of m GOEs. To proceed, we again consider an ensemble of matrices of the form of Eq. (4). Now H_0 is a block diagonal matrix consisting of m blocks whose dimensions are M_i , $i=1, 2, \dots, m$, with $\sum_{i=1}^m M_i = N$. The elements of H_0 have zero mean and variances given by Eq. (3). We define H_1 to be zero where H_0 is nonzero and elsewhere its elements have zero mean and variances given by Eq. (5).

To obtain a formula for the ALD, we note that

$$\overline{\text{tr} H^2} = \sum_{j,k=1}^N \overline{H_{jk}^2} \quad (51)$$

$$= N^2 \sum_{i=1}^m \frac{M_i}{N} \sigma_i^2 \quad (52)$$

with

$$\sigma_i^2 = \frac{1}{4A} \left[\frac{M_i}{N} + \alpha^2 \left(1 - \frac{M_i}{N} \right) \right]. \quad (53)$$

Considering a single line of H , the σ_i^2 consist of a term which is the product of the variance of a single nonzero element of H_0 with the probability for being in block i plus a term which is the product of the variance of a single nonzero element of H_1 with the probability for being outside block i . In the sum in Eq. (52), the σ_i^2 are weighted with the fraction of lines which find themselves in block i .

For the fourth power of H , we find

$$\overline{\text{tr} H^4} = 2 \sum_{j,k,l=1}^N \overline{H_{jk}^2 H_{jl}^2} \quad (54)$$

$$= 2 \sum_{j=1}^N \left[\sum_{k=1}^N \overline{H_{jk}^2} \right]^2 \quad (55)$$

$$= 2N^3 \sum_{i=1}^m \frac{M_i}{N} \sigma_i^4. \quad (56)$$

Although the authors were unable to obtain an equation analogous to Eq. (55) for higher powers of H , Eqs. (52) and (56) strongly suggest that

$$\overline{\text{tr}H^{2s}} \simeq C_s N^{s+1} \sum_{i=1}^m \frac{M_i}{N} \sigma_i^{2s} \quad (57)$$

for large N .

Substituting Eq. (57) into Eq. (12) and again using Eqs. (17), (7), and (19), we obtain

$$\rho(E) = \sum_{i=1}^m \frac{M_i}{N} \rho_1(a_i, E), \quad (58)$$

where ρ_1 is given by Eq. (20) and

$$a_i = a^2 \left[\frac{M_i}{N} + \alpha^2 \left(1 - \frac{M_i}{N} \right) \right] \quad (59)$$

$$= \frac{1}{A} \left[M_i + \lambda^2 \left(1 - \frac{M_i}{N} \right) \right]. \quad (60)$$

From Eq. (58), it follows that the CLD, Eq. (21), is given by

$$x(E) = \sum_{i=1}^m \frac{M_i}{N} x_1(a_i, E), \quad (61)$$

with x_1 given by Eq. (22).

In Fig. 5, we compare Eq. (58), for the ALD with numerical simulations for several values of the α (see figure caption for details) and excellent agreement is obtained.

VI. CONCLUSION

In conclusion, by assuming that terms containing patterns of unlinked binary associations dominate the averages of the

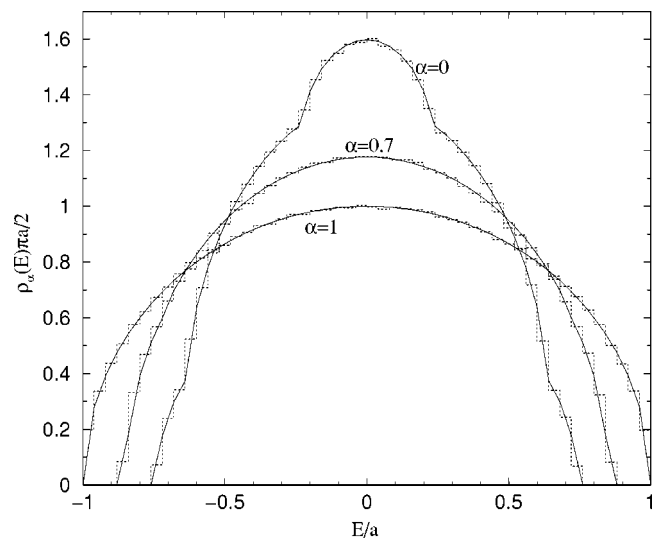


FIG. 5. Graph of $\rho_\alpha(E)$ illustrating the transition 3 GOEs to GOE. The three blocks have size $M_1=50$, $M_2=400$, and $M_3=550$. The densities for $\alpha=0$, 0.7, and 1.0 are shown. The calculations were performed with $A=N/4$, for which the radius of the Wigner semicircle is $a=2$. The solid lines were calculated using Eq. (58) and the dotted histograms by numerically diagonalizing an ensemble of 100 matrices.

traces of powers of matrices, we have derived formulas for the average level density for two deformations of the Gaussian orthogonal ensemble. The first describes the transition from the Gaussian orthogonal ensemble to the Poisson ensemble and the second the transition from the GOE to m GOEs. The formulas obtained are in excellent agreement with numerical simulations.

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